

**Plato´s Neglected Problem and the Property View of Numbers**  
**Back to Euclid with some differences**

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## Abstract

Seldom noted in the philosophy of mathematics is the fact that Plato divides his world of forms or ideas into two different realms, a higher and a lower realm, respectively. In the higher, each form/idea – e.g., a pure number – exists only as a unique entity, but in the lower, each form/idea – e.g., a pure number – exists as a multiplicity. The book notes that such a bifurcation of numbers is not made in the modern philosophy of mathematics, but it argues that the bifurcation needs to be made. Otherwise, the philosophical number analyses made cannot explain why, for instance, there are two distinct number 3s in an addition such as  $3 + 3 = 6$ . The book then argues that the usually neglected *property view of numbers* allows for the bifurcation needed. According to this view, the original natural numbers are properties of collections, and the real numbers are properties of proportion-relations. Furthermore, it is argued that the property view of numbers is better in harmony with the findings of modern empirical numerical cognition research than the traditional philosophical analyses of numbers are. Central to the book is Euclid's old notion of *ratio*, which was put forward in his classic treatise *Elements*; in this book it is called *proportion-relation*. The book's math-philosophical message is: Back to Euclid with some differences. It claims that Euclid's distinction between numbers and magnitudes ought to stage a comeback as a distinction between two number lines, one for the *original* natural numbers and one for the real numbers, and where the original natural numbers cannot be regarded as a subclass of the real. The line for the real numbers contains, however, numbers that are *counterparts* to the original natural numbers.