Order, Direction, Logical Priority and Ontological Categories

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Abstract The ontological problem of sense/order/direction of non-symmetric relations is the problem of how to explain from where the most basic such sense/order/direction comes. The paper argues that, given some ontological presuppositions, one should distinguish between at least three kinds of relation senses: order, direction, and logical priority. The order-sense stems from the un-reducible three-term relation *betweenness*, the direction-sense has its root in the un-reducibility of the phenomenon of intentionality, and the logical priority-sense is a special kind of ontological dependence.

1 Introduction

Another but too long title for this paper could have been: 'How the analysis of what Bertrand Russell calls the "sense" of relations depends on ontological categories; especially those that I find reasonable'. But let me start from scratch: what is the problem or problems that might be called scientific or philosophical problems of order and direction ("sense") of relations? The scientific-technological problems consist in how to construct measuring scales. Disregarding the concrete problems around specific scales, this problem is well stated in the following quotation:

> The concept of order is fundamental. Without such a concept one has only that of *different* classes of things […] In principle, the concept of order is simple enough. If one can place the objects on a line, from left to right, according to the magnitude of the attribute in question, so that object *a* is to the left of object *b* if and only if *aRb*, then the objects have an order relationship. (Hand 2004, pp. 33-34)

A natural-scientific or technological order, i.e., a scale, is created when classes of real or possible objects in the material world have been put into a one-to-one relation to some part of, or to the whole of, the mathematical number line. This problem is not the problem of this paper. Here we shall be concerned with the *ontological* problem of order, the problem of how there can be any order at all. In relation to this, neither the order of the number line nor the order of the classes mentioned can be allowed to be taken as pre-given. Of course, the existence of relations is taken for granted, but the "sense" problem appears whether or not the relations are regarded as being mind-independent or mind-dependent.[1] In both cases the problem is how to isolate the order-feature from other ontological categories or subcategories. In principle, it could have been the case that what is regarded as constituting order and direction is so to speak ontologically neutral, i.e., all different ontological systems proposed by philosophers would give the same answer about what constitutes order and direction. But as will become clear, this is not the case.

The first philosopher to clearly state the ontological problem of order seems to be Bertrand Russell in *The Principles of Mathematics* from 1903; and the philosopher who should be credited for trying to keep it alive even now a hundred years later, is Erwin Tegtmeier (1992, ch. V; 2004); it has never received much attention. The late discovery of the problem is due to the fact that before Russell and some other philosophers of his time, e.g., C. S. Peirce (1931) and G. E. Moore (1960), all philosophers took it for granted that from an ontological point of view *relations* must be reducible to some kind of un-reducible *relational properties* or to Aristotle's "the relative," the *pros ti* (Jansen 2006).[2] Since traditional logic taught that every basic proposition has one subject and one predicate, even propositions that in appearance were about relations had in the ontological end to be reduced to subject-predicate propositions.

On the linguistic level, the distinction between relations and relational properties can be described as follows. The relation*al* predicates 'larger than *b*' and 'smaller than *a*' can be used

[1] However, the relation deniers have to face the order problem in another and even harder way: how can the order in the very useful scales of physics, chemistry, and technology be explained if there are no relations at all? As already Russell said: "I do not believe, for instance, that those who disbelieve in the reality of relations […] can possibly interpret the numerous parts of science which employ asymmetrical relations (Russell 2004, p. 337)." In my paper "Hypo-Realism with Respect to Relations" (Johansson 2012) I have worked out a real defense of the mind-independent existence of internal relations such as *larger than.*

[2] One way to look upon relational properties is to regard them as reducible to conjunctions (or mereological sums) of a relation and a monadic property, but this is an anti-Aristotelian analysis of relational properties. The medievals thought of Aristotle's *pros ti* as accidents that are *pointing to* or has a *being toward* another being; they have an *esse-ad* (Henninger 1989, p. 5).

to make assertions of the form *Fa* and *Gb* such as '*a* is (*larger than b*)' and '*b* is (*smaller than a*)', but they cannot be used to make assertions that have the relation logic form *aRb*, and where in the proposition the relation predicate '*R*' is equally connected to two subject terms, '*a*' and '*b*'. Note the difference introduced between a relation*al* predicate (which is a one-place predicate) and a relation predicate (which is an n-place predicate, n > 1).

If we do not simply hide the copula of '*a* is *F*', as is done in the *Fa* of predicate logic, but write '^{is}Fa', we should write the relation logic formula *aRb* as '$a\ ^{is}R^{is}\ b$'. That is, in *aRb* there is a two-place copula, which means that *R* is predicated of two subjects, *a* and *b*, not only one. Although I have not seen it explicitly stated by Russell himself, his view implies that whereas a relation*al* property predicate needs only the ordinary copula, a two-place relation predicate needs a *two-place copula*, a three-place predicate a three-place copula, and so on.

Now I will quote Russell; in the quotation it is taken for granted that '*a* is before *b*' can be formalized as *aRb*. The quotation is not from the 1903-Russell of *The Principles of Mathematics*, since Russell soon changed one of his views on relations. This we know from a 1913 manuscript of his; posthumously published in 1984. I will soon present the change, but first the quotation where the 1913-Russell explains what the ontological problem of "sense" is:

> Thus the difference which demands elucidation is not that between "*a* is before *b*" and "*b* is after *a*", but that between "*a* is before *b*" and "*b* is before *a*". This fact shows that, in the understanding of the abstract "before", which is what we are trying to isolate, there must be some kind of reference to terms, something, in fact, which we call "sense" or "direction" [or "order"]. The two propositions "*a* is before *b*" and "*b* is before *a*" contain the same constituents [*a*, *b*, before], and they are put together according to the same form [– is before –]; thus the difference is neither in the form nor in the constituents. (Russell 1992, p. 86; however, in the quotation capital letters *A* and *B* have been changed into *a* and *b* in order to make the symbolism of my paper more coherent)

The first assertion in the quotation means that Russell subscribes to *the identity view of converse relations* (Johansson 2011, sect. 2-3). This view holds that the use of the converse predicates 'before' and 'after' makes no difference to what is described; the sentences '*a* is before *b*' and '*b* is after *a*' describe the same state of affairs. The predicates merely affect

where '*a*' and '*b*' should be placed in the sentences. Therefore, the ontological problem of relation "sense" is distinct from the linguistic problem of what constitutes the difference in meaning between '*a* is before *b*' and '*b* is after *a*'. According to the 1913-Russell, and explicitly so, the 1903-Russell is wrong on this issue (Russell 1992, p. 87, n3). A bit further on, the quotation above continues:

> It might perhaps be supposed that every relation has one proper sense, i.e. that it goes essentially *from* one term *to* another. In the case of time-relations, it might be thought that it is more proper to go from the earlier to the later term than from the later to the earlier. And in many relations it might be thought that one term is *active* while the other is *passive*; thus "*a* loves *b*" seems more natural than "*b* is loved by *a*". But this is a peculiarity of certain relations, of which other show no trace. Right and left, up and down, greater and less, for example have obviously no peculiarly "natural" direction. And in the cases where there seemed to be a "natural direction" this will be found to have no logical foundation. In a dual complex, there is no essential order as between the terms. The order is introduced by the words or symbols that are used in naming the complex, and does not exist in the complex itself. Our problem arises from the fact that, although this is the case, a different complex results from interchanging the terms, and that such interchange *looks* like a change of order. [...] We must therefore explain the sense [or order] of a relation without assuming that a relation and its converse are different entities. (Russell 1992, pp. 86-87)

I will soon analyze one by one three of the relations that figure in this quotation: the temporal *before* (converse predicate: 'after'), *loves* (converse predicate: 'is loved by'), and (instead of *greater than*) *larger than* (converse predicate: 'smaller than'), but I will in one section also briefly discuss the logical *before*, the relation *logically prior*. However, first I want to fasten attention to what Russell is presupposing when he formulates the problem; as a problem-solver we will not return to him, since he never returned to the problem. There are three obvious ontological presuppositions, and these will henceforth be presuppositions for my discussion, too.

First, the falsity of extreme nominalism, the view that all non-linguistic particulars are bare particulars, and that therefore all structure comes from language. Russell takes it for granted that there are at least property instances (of monadic universals) or simple tropes in the world;

and, furthermore, that there can be meaningful talk of relations that directly relate such instances or tropes to each other.

This means, second, that he regards the view that relations can be nothing but set-theoretical constructs (i.e., sets of ordered pairs) as being false, too. His own view was that the relations at issue primarily are *universals*, but some have claimed that they are only simple *tropes* and some that they are *ontological pseudo-entities* (see Johansson 2011, sect. 3). The discussion in this paper is meant to be neutral between these three views, even though I think there are both relational universals and relational instances (i.e., complex tropes) (Johansson 2009), and agree with Herbert Hochberg's statement: "In rigorous ontology, nothing is free—if it is a 'pseudo-entity' then one should either not talk about it or not employ it in one's analysis (Hochberg 2004, p. 39)."

Furthermore, it doesn't matter to the discussion whether the relations are regarded as being mind-independent (be it relational universals, relational instances, relational tropes, or relational pseudo-entities) or mind-dependent (conceptual and perceptual relations). The problem of "sense" is there anyhow.

Third, when relata are property instances or simple tropes there are no converse relations, only converse predicates, i.e., as I have already said, the 1913-Russell presupposes the identity view of converse relations. Obviously, the relations of set theory have converses; necessarily, whenever *a* and *b* are distinct entities, the ordered pair <*a*, *b*> is distinct from the conversely ordered pair <*b*, *a*> (Johansson 2011, sect. 3).

The questions (i) 'Are there relations or only relation predicates?' (the problem of the existence of relations), (ii) 'Are there converse relations or only converse relation predicates?' (the problem of the identity of converse relations), and (iii) 'Do non-symmetric relations have a "sense"?' (the problem of order), should be kept distinct, even if the way one answers one of the three problems has a bearing on how one can consistently answer the other two. To my mind, these problems are sometimes in an unhappy way conflated.[3]

All the relations that I shall analyze are non-symmetric relations, but all of them are not asymmetric relations. The relation *loves* is in the terminology of the 1903-Russell a *not*-symmetric relation (Russell 2006, p. 25). Using modal notions as primitives, and allowing semi-formal expressions ('if *p* then *q*' means informally 'if *p* is true then *q* is true'), symmetric relations can be defined as follows: Symmetric relations $=_{def}$ necessarily, *aRb* iff

[3] I think this is true of some parts of Reinhardt Grossman's defense of relations (1983, 1992) and of the papers (Fine 2000, 2007), (MacBride 2007), (Newman 2002), and (Tegtmeier 2004). I discuss problem (iii) in this paper, problem (ii) in (Johansson 2011), and problem (i) in (Johansson 2012).

bRa. Non-symmetric relations are then contradictory opposites, and asymmetric relations contrary opposites. The definitions and their relationships can be displayed as follows:

Non-symmetric relations

¬ (necessarily, *aRb* iff *bRa*)

Asymmetric relations

necessarily, if *aRb* then ¬ *bRa*;

example: necessarily, if (*a* is larger than *b*) then ¬ (*b* is larger than *a*)

Not-symmetric relations

not-necessarily, if *aRb* then ¬ *bRa*;

example: not-necessarily, if (*a* loves *b*) then ¬ (*b* loves *a*)[4]

2 Analysis of the Relation *Larger Than*

When *a* and *b* are property instances or simple tropes,[5] the relation *larger than* referred to in the sentence '*a* is larger than *b*' is an internal relation in David Armstrong's sense (1997, ch. 6.2). That the relation *R* in *aRb* is internal means: necessarily, if both *a* and *b* exists, then *R* is instantiated or exemplified.[6] One might also say: in internal relations the relata do collectively entail the relation, in external they do not. In my opinion, the usual examples of internal and external relations have a feature in common that makes them differ from some ontologically even more peculiar relations such as *inherence*, *instantiation*, and *ontological dependence*; the last relation will pop up in Section 5. In what follows, by stipulation, I will regard all internal and external relations that I speak of as being such that, in principle, both

[4] The last definition can also be stated thus: if *aRb*, then ((possibly, *bRa*) & (possibly, ¬ *bRa*)). Example: if (*a* loves *b*), then ((possibly, *b* loves *a* & possibly, *b* does not love *a*)).

[5] For the sake of stylistic simplicity, the term 'property instance' is in the paper understood in such a wide sense that even philosophers such as David Armstrong (1978, 1997) and Reinhardt Grossmann (1983, 1992) can be said to accept the existence of property instances. Both deny the existence of what is normally called instances, but Armstrong posits a relation of *instantiation* and Grossmann posits a relation of *exemplification*. That is, I allow myself to say that there is a property instance where Armstrong has a state of affairs consisting of the union (*instantiation*) of a monadic universal and a particularity that "is closer than relation" (1978, II: p. 3), and where Grossmann has a spatiotemporally localized individual that *exemplifies* a completely non-localizable property universal. Property instances may be called *complex* tropes, since they are property particulars that in some sense contain a universal and so are not simple. For my own detailed views on universals and particulars, see Johansson 2009.

[6] Note that whether or not *larger than* is an internal relation depends on what its relata are. If *a* and *b* refer to things that can change their size, then the relation referred to in '*a* is larger than *b*' is an external relation.

the *relata* can exist independently of each other; a corresponding spatiotemporal *relation R* (an instantiated universal, a trope, or a pseudo-entity) can of course not exist independently of either of them. If *a* disappears without substitution (but not *b*) so does the spatiotemporally existing or pseudo-existing *R*; and if *b* disappears without substitution (but not *a*) so does this *R*. In order for the relations *larger than*, *loves*, and *before* to exist spatiotemporally, there have to be, so to speak, both an *a* and a *b*, even though the existence of *a* does not imply the existence of *b*, or vice versa.

The sentence '*a* is larger than *b*' can be used not only to describe a relation between two spatiotemporal entities, but also to describe a relation between two abstract entities such as two mathematical numbers. But this does not affect what was said in the preceding paragraph. If this square instance or square-trope □ is called '*a*', and this one □ '*b*', then there is here (in the perceptual field of the reader) a case of the relation *larger than* between *a* and *b*; but if one of *a* and *b* suddenly disappears, there is here no longer any case of the relation *larger than*. On the other hand, if we regard these two squares as being only representations of two corresponding abstract purely geometrical-mathematical squares, then the problem of disappearance does not arise.

What then about the Russellian "sense" (direction, order) of the internal relation *larger than*? On the identity view of converse relations, the assertions '*a* is larger than *b*' and '*b* is smaller than *a*' must describe one and the same state of affairs, say this fact: □ □ .[7] Is it then really true, as another identity view theorist explicitly has it, that all order and direction "is connected with the order in which we *think* of the terms of the relation (Segelberg 1999, p. 190)"? I think not, even though I am of the opinion that there is in the relational fact above (under the description given) no fourth constituent besides *a*, *b*, and *larger than*.

The solution to the problem of "sense" lies here in realizing that we automatically interpret the two-place relation predicate 'larger than' as referring to an asymmetric *transitive* relation, and that asymmetric transitivity needs for its definition a third term: Transitive relations $=_{def}$ necessarily, if *aRb* & *bRc* then *aRc*. That is, to make an analogy with Russell's claim about relational predicates: as the one-place predicate 'larger than *b*' is, when used in an assertion, referring to a two-term relation *larger than*, the two-place predicate 'larger than' is, when used in an assertion, referring to a relation that is at least a three-term relation. In a world with only two size instances or simple tropes, a speaker can use the relation predicate 'larger than'

[7] It doesn't matter whether the fact or state of affairs is regarded as a concrete spatiotemporal entity or an abstract non-localizable entity á la R. Grossman (1992, p. 87).

only if she can imagine a third size instance or simple trope that is not equal to the size of any of the two really existing sizes.

In the two true assertions '*a* is larger than *b*' and '*b* is smaller than *a*' a certain relational fact (□ □) is explicitly described, but the relation predicates used implicitly bring in a reference to something third, which from the speaker's and the listener's epistemological point of view belongs to the background knowledge, and from the semantic point of view belong to their linguistic context. What is it? Answer: it is either a metric size scale or a corresponding informal ordinal scale. Now, since out of every metric scale an ordinal scale can be abstracted, I will in what follows only discuss the ordinal scale. The assertions place the objects *a* and *b* in a context where it is taken for granted that there are other, real or imagined, size instances or tropes with which *a* and *b* can be compared and regarded as being larger and smaller. And to realize this makes quite a difference to the analysis.

An ordinal size scale (in set-theoretical terms a complete/connected strict partial ordering) contains for every triad of different size values, *a*, *b*, *c*, the three-term relation *betweenness-in-size* referred to in the sentence '*b* is between *a* and *c*', *Babc*, used above. The formal features of all kinds of *betweenness* are (Gärdenfors 2000, pp. 15-17):

(0) if *Babc*, then *a*, *b*, and *c* are distinct points;
(1) if *Babc* then *Bcba*, i.e., if *b* is between *a* and *c*, then *b* is between *c* and *a*;
(2) if *Babc* then ¬ *Bbac*, i.e., if *b* is between *a* and *c*, then *a* is not between *c* and *b* (rules out betweenness on a line that is a loop);
(3) if *Babc* & *Bbcd* then *Babd*, i.e., if *b* is between *a* and *c* and *c* is between *b* and *d*, then *b* is between *a* and *d* (the consequent can just as well be *Bacd*);
(4) if *Babd* & *Bbcd* then *Babc*, i.e., if *a* is between *b* and *d* and *c* is between *b* and *d*, then *b* is between *a* and *c* (the consequent can just as well be *Bacd*).

Such a relation of *betweenness* appears everywhere where there is a relation predicate that combines asymmetry with transitivity. If ((*R* is transitive and asymmetric) & (*aRb* & *bRc*)), then *Babc*. But there is also an implication that goes the opposite way. Where there is a difference in a certain respect (e.g., size difference) between three terms and a case of the relation *betweenness* between them, there is transitivity and asymmetry. If *Babc*, then a two-place relation predicate '*R*' that is transitive and asymmetric can be construed; in set theory, from {*a*, *b*, *c*} and *Babc* we can construe $R = \{\langle a, b\rangle, \langle b, c\rangle, \langle a, c\rangle\}$.

However, the three-term relation *Babc* cannot be derived from a conjunction of descriptions of three two-term relations *Rab*, *Rbc*, and *Rac* that contain no information about transitivity. For instance, from '*a* and *b* are different in size', '*b* and *c* are different in size', and '*a* and *c* are different in size' it is impossible to derive '*b* is between *a* and *c* in size'. Putting the last two paragraphs together, we reach this conclusion: *betweenness* is an un-reducible three-term relation.

Long ago, Charles Sanders Peirce claimed that all (or all "genuine") three-term relations are un-reducible to (i.e., have a "higher nature than," or cannot be "analyzed" into) two-term relations, but that all n-term relations with n > 3 are reducible to two- and three-term relations (Tiercelin 2010; Peirce 1931-58, 1.363, 6.323, 7.537). His examples, however, have another character than the internal relations above; he exemplifies with actions where a person (*a*) gives or sells something (*b*) to someone (*c*). Both Peirce's general un-reducibility thesis and his general reducibility thesis are as such interesting, but they need not be solved in order for me to proceed with the problem of order (which I think Peirce never discussed); I need only the un-reducibility of *betweenness*. Let me nonetheless note, that it seems quite possible to analyze an ordinal scale with *n* values into *n* minus *2* number of *betweenness* relations. This follows from the third formal feature of *betweenness*: (3) if *Babc* & *Bbcd* then *Babd*.

If, as I will now take it for granted, *betweenness-in-size* between property instances or simple tropes is an un-reducible three-term internal relation, then, in contradistinction to two-term internal relations, it contains an order. The fact that necessarily one term is in the middle (the second formal feature), is an order inherent in the relation. It would be wrong though to say that *betweenness* contains a direction, since what is in the middle can be approached from either side; even though, looking at a whole scale, it may in some sense be more natural to move from the zero point upwards than from a certain point downwards; especially if the scale like those of length and mass has no upper limit.

In the second quotation above, Russell writes: "In a dual complex, there is no essential order as between the terms." I agree when the relation in the complex is an internal relation,[8] but then, in contradistinction to Russell, I add: but in some *triple complexes*, there is an essential order as between the terms. Look at the following two three-objects facts: (i) □ □ □ (*a*,*b*,*c*) and (ii) □ □ □ (*a*,*c*,*b*). One aspect of both facts can be described by one and the same sentence: 'with respect to size, *b* is between *a* and *c*'. It can also be described with the content-equivalent sentences '*a* is more size-like *b* than *c*' and '*c* is less size-like *a* than *b*'.

[8] I regard *causality* as an external relation.

Russell claimed that some ordinary assertions that appears in the form '*a* is *F*' really have the structure *aRb*; I am claiming that some ordinary assertions of the form '*a* has the relation *R* to *c*' does not have the logical structure *aRb* (or *Rab*), but *Rabc*. Ordinary language puts much in the context.

To be fair to Russell, I would like to add that he himself in a way foreshadowed my conclusion. The 1903-Russell (Russell 2006) says in § 188 that "The notion of order is more complex than any hitherto analyzed. Two terms cannot have an order, and even three cannot have a cyclic order. Owing to this complexity, the logical analysis of order presents considerable difficulties," and then in § 208 he states that "We have now seen that all order depends on *transitive asymmetrical* relations" (italics added). But some paragraphs later, § 217, transitivity is gone and he says simply: "We have now seen that order depends upon asymmetrical relations." But with respect to internal relations, asymmetry alone does not create order, and neither does transitivity alone (since symmetric relations are transitive, too); both are needed.[9]

Conclusion: the "sense" of the relation in the state of affairs *a is larger than b* comes from the background knowledge that *a* and *b* are implicitly related also to the three-term relation *betweenness-in-size*, which really has an in-built order (but no direction). It can also be put like this: there is with respect to property instances or simple tropes no asymmetric and transitive purely *two*-term relation *larger than*, only a two-place relation predicate 'larger than'.[10]

3 Analysis of the Relation *Loves*

One result of the preceding analysis can be stated thus: what smacks of "sense" in the presumed two-term internal relation *larger than* is imposed from the outside, from a three-term relation. A similar conclusion, but with the opposite twist, will be reached in the analysis of the relation *loves*: in the state of affairs described by '*a* loves *b*' and '*b* is loved by *a*' (*aRb*), there is in the relation *R* no "sense." Again, what smacks of "sense" comes from the outside, but this time from, and only from, the relatum *a*.

[9] The set-theoretical notion of strict partial ordering is, by the way, defined as being both asymmetric and intransitive.

[10] In (Johansson 2004b) I make an analogous claim about the predicates of some parthood relations (e.g., functional parthood). In natural languages they may seem to refer to two-term relations (e.g., the *F*unction of the *h*eart is to pump blood in the *b*ody, *hFb*), but as a matter of fact there is implicitly a three-term relation.

Obviously, there can be no loving relation as described by '*a* loves *b*' if the person *a* does not exist. But what about the case where *a* exists but *b* does not, and (which is common in the relation literature) we take it for granted that we can formalize the love state of affairs as *aRb*? Since *R* cannot exist spatiotemporally if not both *a* and *b* so exist, the relation of *loving* symbolized by '*R*' cannot exist if *b* does not exist. But of course, on the other hand, if *a* loves *b* and *b* unbeknownst to *a* has died, nothing in *a*'s intentional state has changed. That is, in some sense of '*a* loves *b*' the loving is still there, even if it cannot exist in a veridical perception, only in intentional states and acts such as thinking, imagining, and dreaming.[11]

The simplest way to solve the puzzle that there seems both to be and not to be a relation *loving* in the *b*-has-died situation, is to distinguish between on the one hand *a*'s *intentional state* of loving *b* (where intentionality is taken to be an un-reducible phenomenon understood in about the sense expounded by Franz Brentano (1995 [1874]), Edmund Husserl (1970 [1901]), and John Searle (1983)), and on the other hand the *internal relation* of loving that is entailed by the existence of the intentional state of loving in *a* and the simultaneous existence of the beloved one *b*. Let me call the former '*I-love*' (= the *I*ntentional state of loving *b*, which is logically independent of the existence of *b*) and the latter '*R-love*' (= the internal *R*elation *loving*, which is logically dependent on the existence of both the *I-love* and *b*). It is the *I-love* of *a* that is the real relatum in *aRb*. If we symbolize the *I-love* as *Ia*, we can exchange *aRb* for *(Ia)Rb*.

When *b* unbeknownst to *a* dies the *I-love* remains, but the corresponding *R-love* disappears, since it requires the existence of both the relata, *(Ia)* and *b*. It is as simple as that. Now, of course, there is a sense of 'love' in which it can be said that a person *a* consciously loves a person *b* who is known to be dead, but that is a qualitatively different kind of intentional state; one where the intentional object is no longer taken to be a real existent, but a merely imagined object. In such cases, no *R-love* can possibly exist. The common impression that *loves* must be an external relation with a direction comes, I think, from the common but philosophically unfortunate fusion of *I-love* (an intentional state) and *R-love* (an internal relation).[12]

[11] Since veridical perceptions are left out of account, nothing at all will below be said about the problem whether disjunctivism is true or false.

[12] In my opinion, this fusion is what flaws (compare note 3) R. Grossmann's in some respects good defense of the existence of relations (1983, ch. 3; 1992, pp. 51-57). The flaw is made clear by Laird Addis (2010), who makes a general distinction between *intentional states* and an *intentional relations* that seems to be more or less the same as the one I have used when distinguishing between *I-love* and *R-love*; even though he does not characterize intentional relations as being some kind of internal relations. I have earlier made the same kind of

In the light of the preceding sections I now claim, without further arguments, that the two-term internal relation *R-love* contains no "sense," but that the *I-love* of the relatum has a "sense." Moreover, the latter has a clear-cut direction; *a*'s intentional state of loving is directed at *b*. All intentional acts and states have so to speak a from-to structure. There is a from-pole located in the subject plus a directedness towards a to-pole (the intentional object; which may or may not exist). Therefore, the conclusion of this section is: the two-place relation predicate 'loves' has its "sense" due to the fact that the order-less and direction-less two-term relation *R-love* is not kept distinct from the directedness of the *I-love* that makes up the first relatum. Often, quite an abstractive effort is needed in order to isolate features of an internal relation from features of its relata.

As an aside, it can be noted how well the unanalyzed relation *loves* fits "the relative" of the medieval relation(al property) realists. Duns Scotus says: "A real relation is not a being *per se* [i.e. a substance], nor an interval between two extremes, nor in two as in one subject, but [is] in one [extreme] and toward another" (quoted from Henninger 1989, p. 68). Such a relation(al property) is in the Aristotelian sense an accident that inheres in one of the relata, but it is an accident that is "toward another" substance or accident (compare note 2). In the terminology of Scotus, Russell's (and mine) view is that a two-term relation is "in two as in one subject."

4 Analysis of the Temporal Relation *Before*

There are quite a number of different ontologies of time around in contemporary philosophy. And if one does not want to subscribe to one specific such ontology (which is my intention in this paper), the asymmetric temporal relation *before* has to be discussed *at least* two times: what it looks like when a future directed time's arrow is posited, and when the existence of such an arrow is denied. Let me say a few words about this dichotomy before proceeding to the two corresponding analyses of *before*.

When presentism (the view that only the content of "the now" exists) and the so-called 'the growing universe theory' (also the past is ascribed some kind of existence) posit a time's arrow, this arrow is in the now directed towards as yet not existing future time points; this is what I will call 'the time's arrow'. However, eternalism and four-dimensionalism (which

distinction in a critical remark on Searle's analysis of intentionality (Johansson 2003). In that paper, what Addis calls 'intentional relations', I call 'relations of satisfaction (of intentional states)'.

deny the existence of an objectively privileged now, and claim that all time points exist in the same way and so to speak eternally) can try to posit a *counterpart* to what I have baptized the time's arrow.

If the truth of the second law of thermodynamics is taken for granted, then eternalism and four-dimensionalism have recourse to a physical process that is directed towards *increasing* entropy, but this entropy-arrow can in my opinion not in any sense be called the time's arrow. Rather, the law takes the direction of time as pre-given. If, instead, objective time is assumed to run from the future towards the past, then the law has to be reformulated into a law of *decreasing* entropy.

Another attempt to find a counterpart arrow can be associated with Hugh Mellor (1998). It consists in trying to ground the direction of time in an asymmetric non-temporal causal relation, a causality-arrow one might say. Like the causal relation accepted by Hume, there is nothing in such a causality-arrow that literally involves bringing into being or producing the effect, since for Mellor (but not Hume) both the cause-event and the effect-event exist eternally. But in another respect Mellor turns Hume upside down. Hume defines 'causality' partly by means of 'before' (or 'succession'), whereas Mellor defines 'before' by means of 'causality'. Now, I will simply leave Mellor's and similar views on the temporal relation *before* out of account. In fact, if I am right below, there is a way in which eternalists and four-dimensionalists can introduce *temporal order* (but not *direction*) without bringing in either thermodynamics or causality.

Let me start with the analysis of the *non-arrow before*. Here, my point is quickly made. The analysis of *larger than* in Section 2 is an analysis of *non-arrow larger than*,[13] and it has such a general nature that it applies to *non-arrow before*, too. In the purely dual temporal non-arrow state of affairs *a is before b* there is no "sense." If a "sense" nonetheless is apprehended, it must come from the three-term relation *temporal betweenness* that hovers in the background. The three-term relation *b is between a and c in time* has an in-built (but direction-less) order. In other words: there is no asymmetric and transitive *two*-term relation *non-arrow before*, only the two-place relation predicates '(non-arrow) before' and '(non-arrow) earlier than' with the corresponding converse predicates '(non-arrow) after' and '(non-arrow) later than'. Now, perhaps, the ordinary predicate 'before' is often understood as by definition having a meaning that essentially involves directedness, but the predicate '(non-arrow) before' has no such meaning.

[13] In physical-mathematical words, it is an analysis of a scalar magnitude, not a vector magnitude.

When *before* is regarded as being essentially connected with the time's arrow, i.e., we have the relation *arrow before*, another possibility of interpreting the "sense" of *before* arises. Instead of the *betweenness* relation discussed in Section 2, we will now find *intentionality* as discussed in Section 3.

Since the future towards which the time's arrow is directed does not yet exist, the now has to be ascribed the capability of being directed towards non-existent temporal points.[14] This makes the now similar to intentional phenomena such as the loving of the non-existing person *b* discussed in Section 3. Famously, Brentano claimed that intentional directedness is a necessary and sufficient condition for something to be mental, and Husserl countered that it cannot be a necessary condition; there are mental phenomena that lack intentionality. But if there is a mind-independent time's arrow, intentional directedness cannot even be a sufficient condition for something to be mental. Like the state of loving a non-existent person (and like the belief in a completely false assertion or the entertaining of an assertion that is about a fictional entity), the time's arrow is directed towards something non-existent, namely the future.

This similarity means that in ontologies with a time's arrow the state of affairs that *a comes before b* should be analyzed more or less the way *a loves b* has above been analyzed when *b* is dead. More precisely, the sentence '*a* comes before *b*' should first be re-written into '*a* is/was/will-be in a temporal point (t_1) that is/was/will-be directed towards the temporal point (t_2) where *b* exists'. And then it can be claimed that the relatum t_1 has an inherent directedness towards t_2.

Above, I have taken one of the mainstream assumptions of the contemporary philosophy of time for granted, namely that the objective now is absolutely punctual. However, if the perceptual so-called 'specious present' (a duration perceived as both present and extended[15]) is assumed also to have a mind-independent kind of existence, i.e., the objectively existing now is assumed to have a certain small temporal extension, then one might argue that the time's arrow exists *within* each such extended now, too. The earlier parts of the now would then be directed towards the later parts; and even this directedness can, I think, be regarded as a case of intentionality. Let me briefly explain this speculation of mine.

On the mainstream assumption of a punctual now, the assumed intentional directedness of the time's arrow can only be directedness towards non-existent intentional objects (the as yet

[14] In a derived sense, this feature can then be ascribed to past and future temporal points, too.

[15] Defended by, for instance, E. Tegtmeier (1997, §51).

non-existing future time points). However, on the assumption of an extended now, there might *also* in the earlier parts of the now be an intentional directedness towards the later parts of the same extended now. Such a time's arrow has to be compared with the love that is a loving of an existing person.

Assume that the two time points t_1 and t_2 both exist in the same extended now. This means that t_1 comes before t_2 in two ways. First, there is the relation *arrow before*, i.e., t_1 contains the intentional directedness of the time's arrow towards t_2. Second, there is also a direction-less temporal relation *non-arrow before* between the existing relata t_1 and t_2. The analogy with Section 3 is this: the *arrow before* corresponds to the *I-love*, and the *non-arrow before* corresponds to the *R-love* there spoken of.[16]

5 Analysis of the Logical Relation *Before*

In Section 1, I mentioned the fact that the identity view of converse relations is true for relations between property instances or simple tropes, but necessarily false for relations as they are defined in set theory. This means that set theorists cannot without further ado project set theoretical truths about relations onto relations between property instances or tropes. In particular, I would like to point out that Kuratowski's famous set-theoretical definition of the ordered pair (and other such proposed definitions) by no means solves the ontological problem of order or "sense" by *reducing* what has order, ordered pairs, to something that has no order, sets. However, his definition supplies us with an example of a relation that has another kind of relation "sense" than those discussed so far.

Kuratowski's definition is this: $\langle a, b\rangle =_{def} \{\{a\}, \{a, b\}\}$. There is of course no order neither *in* the sets {a} and {a, b} nor *between* them in the set {{a}, {a, b}}, but there is an inevitable logical order between the two sets {a} and {a, b} on the one hand, and the set-of-sets {{a}, {a, b}} on the other. Sets can exist (be construed) even if no sets-of-sets exist (are construed), but not vice versa. Sets are *logically prior* to sets-of-sets; and the sentences '*a* is logically prior to *b*' and '*b* is logically posterior to *a*' describe a state of affairs whose relation constituent has Russellian "sense." This "sense" cannot be grounded in any kind of intentional state, since there is no directedness from sets to sets-of-sets, or vice versa. Furthermore, the

[16] Note, though, that whereas the predicate 'R-love' is not-symmetric, 'non-arrow before' is asymmetric; the time's arrow cannot possibly on ordinary assumptions be directed from t_2 to t_1, too.

"sense" exists whether or not there are any sets-of-(sets-of-sets) and a relation of *betweenness*, i.e., that sets-of-sets exist between sets and sets-of-(sets-of-sets). In all the examples of *aRb* discussed in the preceding sections, the relata *a* and *b* have been assumed to be able to exist independently of one another, but this is exactly what is denied in assertions such as '*a* is logically prior to *b*'. Here, it is claimed that *b* cannot possibly exist if *a* does not exist.

That sets are logically prior to sets-of-sets is a fact independently of whether sets are regarded as Platonic entities or as conceptual constructions; even constructions can have a nature, i.e., constructivism does not imply conventionalism. Already Plato introduced a relation of logical priority. The highest idea in the realm of ideas, the Good, was regarded as a logical presupposition for all the other ideas, but not vice versa. Although denying the existence of Platonic ideas, Aristotle renounced a relation of logical priority. Here comes a quotation: "For isolated entities are logically prior to the same entities in combination (Aristotle 1907, app. D, M, 2, 1076b)." In contemporary analytic ontology, this relation must be regarded a special case of ontological or existential dependence. For overviews, see Lowe (2005) and Correia (2008); my own detailed views are in (Johansson 2004a, ch. 9).

The basic notion of existential dependence can be captured by locutions such as '*x* cannot exist unless *y* exists' (Correia 2008, p. 1014) and 'necessarily, *x* exists only if *y* exists' (Lowe 2005, sect. 1). In what follows, I will restrict the variables to abstract entities; both because of the example chosen, and because this stipulation simplifies things. The fact that there is one and only one abstract entity of each kind (a certain set, a certain universal, a certain mathematical number, etc.), makes some distinctions such as those between specific, generic, essential, and identity dependence superfluous.

Correia's and Lowe's characterization of existential dependence leaves it an open question whether the dependence at hand is mutual or one-sided. However, as *symmetric* internal and external relations can have no order, symmetric or mutual existential dependence can have no order either. Therefore, a relation of asymmetric or one-sided existential dependence has for reasons of completeness to be introduced, too. Lowe presents such a definition proposal, but (because of his special interests) he dismisses it. However, it is highly relevant in the present context, and conforms to earlier ideas of mine (Johansson 2004b, p. 131). A notion of asymmetric or one-sided existential dependence can be defined as follows (compare Lowe 2005, sect. 3):

> *x* depends one-sidedly on *y* $=_{\text{def}}$ (i) necessarily, *x* exists only if *y* exists, and (ii) it is *not* the case that, necessarily, *y* exists only if *x* exists.

Applied to my example, it says: necessarily, sets-of-sets exist only if sets exist, and it is *not* the case that, necessarily, sets exist only if sets-of-sets exist. That is, sets-of-sets have the relation *logical posterity* to sets; or, conversely (but equivalently, given the identity view of converse relations presented in Section 1), sets have the relation *logical priority* to sets-of-sets.

The two-term relation *logically posterior* or *one-sided existential dependence* comes as much with an inherent "sense" as the three-term relation *betweenness* does. Therefore, all ontologists who accept this ontological dependence relation have hereby recourse to still another ontological source of the relation "sense" that Russell once made problematic.

6 Ordinators and Ontological Categories

As noted by Erwin Tegtmeier (1992, p. 186), the ontological problem of order or "sense" does not disappear even if it would be true that there is no order outside of the linguistic realm, i.e., that all order comes from the way we think, talk, and use symbols. We then have to ask from where the temporal and spatial orders of our sentences come: "Yet, a temporal succession of two signs is just another relational fact whose relata need a ground of their order, too (Tegtmeier 2004, p. 158)." I have already presented my views on temporal relations. With respect to spatial relations I will now, in the light of the earlier sections, state my views.

There are two ways in which spatial "sense" can come about. First, in intentional states of perception there is a primitive directedness going from the perceiving subject, the from-pole of the intentional state, out into every perceivable direction in the perceptual field. In veridical perceptions, a subject can have intentional states directed at objects at a spatial distance. One might say that the subject is directed towards other places as the now in traditional presentism is directed towards other and future time points. Second, taken in complete abstraction from perceptions, points on a spatial line can have an order that is grounded in the three-term relation *spatial betweenness*.[17]

Tegtmeier's own solution (1992, ch. V; 2004), however, takes another route. Probably, this is due to the fact that he, in his general philosophical ontology, postulates neither un-reducible

[17] I have consciously restricted my discussion to one-dimensional *betweenness*. We speak also, for instance, of orders between spots on a surface and things in the three-dimensional space, but the question whether or not such orders can be reduced to a number of one-dimensional orders goes beyond the topic of this paper.

intentionality nor un-reducible three-term relations; see (Tegtmeier 1992; 1997) and my remarks in (Johansson 2001). He argues that all order and direction comes from something he calls 'ordinators'. These belong, he claims, to a category *form*, not to the category *thing*. He writes:

> The entities grounding the order in relational facts […], the ordinators, as I named them, belong to the secondary forms. In relational facts they form things which are preformed as individuals or as universals of a certain type. Ordinators are firstness, secondness, thirdness and fourthness. (Tegtmeier 2004, p. 156)

These ordinators he regards as being the ordinary ordinal numbers, if only these are kept completely distinct from the cardinal numbers.[18] Now, I will not here start a discussion about Tegtmeier's ordinators in relation to his own ontological system, but I would really like to say what they look like from my perspective. I agree with Tegtmeier (and so should Russell) that there must be something that deserves at least the label 'ordinator', but then come the differences. Whereas Tegtmeier thinks that this label directly refers to something in the world, I think the label is nothing but a generic term for what I would like to call: (i) 'the *order* inherent in the three-term relation *betweenness*', (ii) 'the *direction* inherent in intentional phenomena', and (iii) 'the *logical priority* that belongs to the essence of the relation *one-sided existential dependence*'.

Whereas Tegtmeier comes to the conclusion that all order and direction ("sense") has one and the same source, ordinators, my view is that there are at least three different kinds of ordinators. I say 'at least', since I will leave as open the question whether or not it is possible to analyze the order of the ordinal numbers by means of *betweenness*, *intentionality*, or *one-sided existential dependence*.[19]

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[18] Personal communication.

[19] My hunch, though, is the following. If the ordinal numbers are essentially associated with a succession operator, and the cardinal numbers with the size of sets (Moore 2002, pp. 125 and 152), then ordinality might be a case of one-sided existential dependence, and the order of the cardinal numbers be grounded in a relation *betweenness-in-set-size*. Ordinal numbers and cardinal numbers ought to have their order relations differently grounded, since in the transfinite realm numbers with different ordinality can have the same cardinality.

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